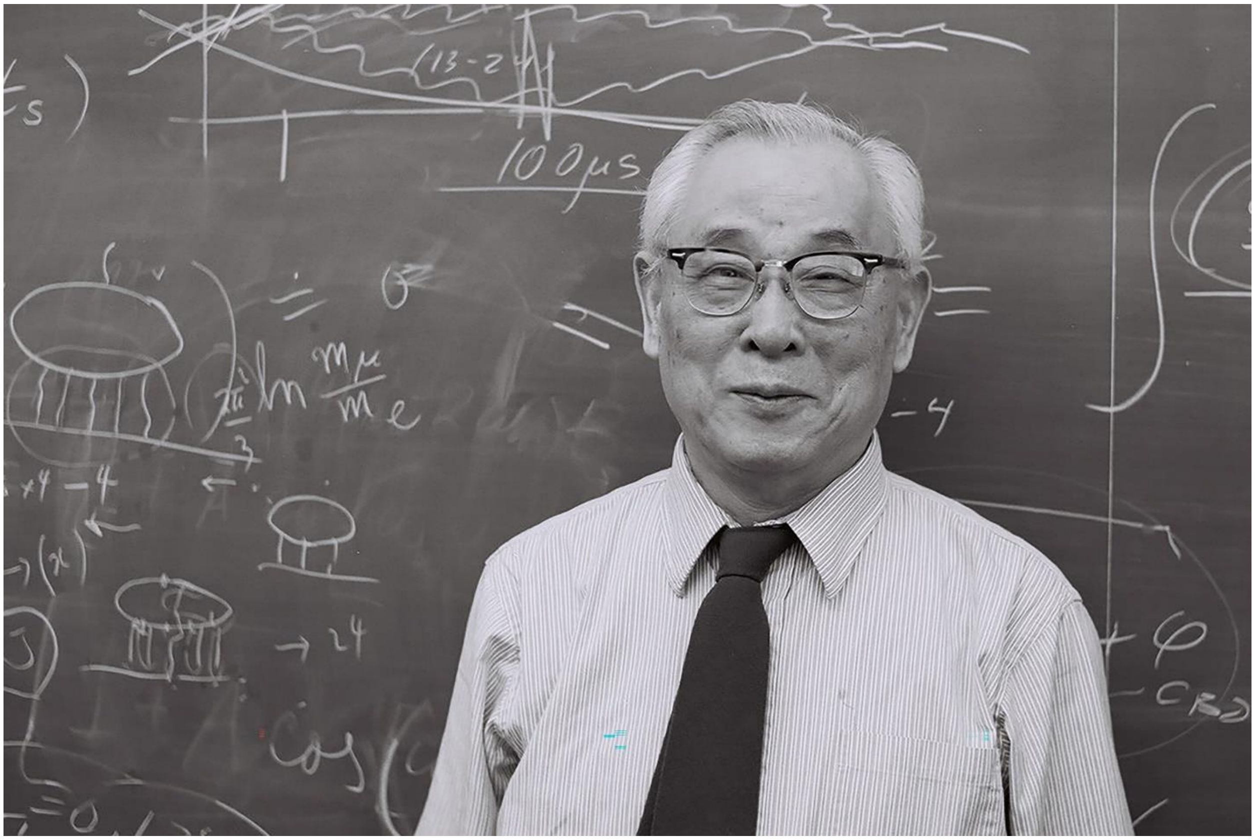
Teacher notes

A spectacular calculation - In memoriam Toichiro Kinoshita (1925-2023)

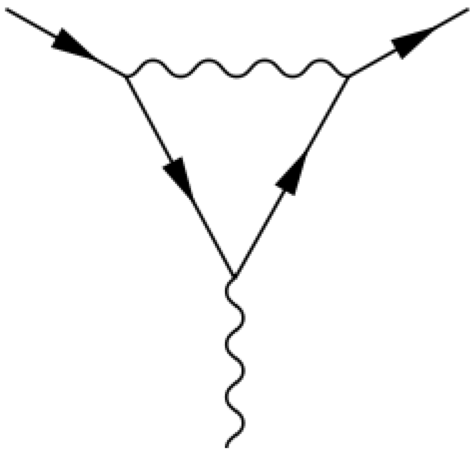


In the teacher note on Antiparticles we saw that the Dirac equation predicts spin for the electron. An electron has electric charge, so a spinning electron has a magnetic moment given by $\vec{\mu} = g\frac{e}{2m}\vec{S}$. The electron behaves as a tiny magnet.

According to the Dirac theory, $g = 2$. However, quantum corrections (i.e., interactions of the electron with virtual photons) provide corrections to g.

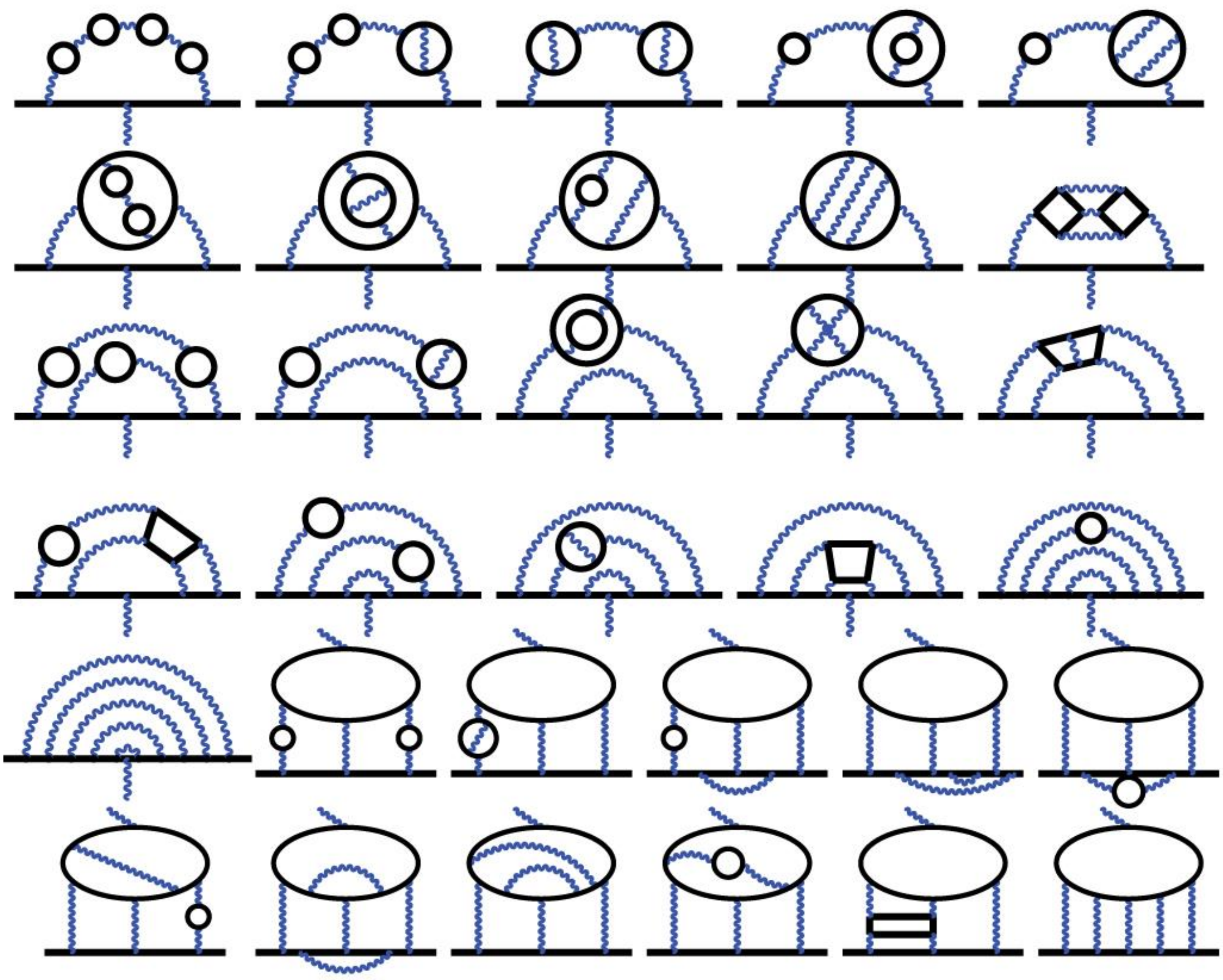
The anomalous magnetic moment of the electron is defined by $a = \frac{g-2}{2}$.

The first quantum correction is obtained by calculating one Feynman diagram:



This diagram contains one loop and so involves one 4-dimensional integral. This was calculated by Julian Schwinger in 1948. His result was that $a = \frac{\alpha}{2\pi} = 0.001\,161\,4$. Here α is the fine structure constant $\alpha = \frac{e^2}{4\pi\varepsilon_0 \hbar c} = \frac{1}{137.035\,999\,084}$.

Additional corrections involve similar Feynman diagrams with more loops. Two loop diagrams give corrections of order α^2 , three loop diagrams contribute corrections of order α^3 and so on. The anomalous magnetic moment of the electron has been calculated to the level of 5 loops i.e., of order α^5 . Each of the 5 loops corresponds to a 4-dimensional integral. Each integral may produce infinities at both the lower and upper limits of integration creating a nightmarish situation of keeping control of what is finite and what is infinite and ensuring that the infinities cancel leaving behind a finite result. There are hundreds of diagrams to be calculated. A few are shown below. Each diagram is representative of a class of diagrams.



The result of the calculation by Kinoshita and his collaborators is:

$a = 0.001\ 159\ 652\ 181\ 643\ (764)$

The experimentally measured value is:

$a = 0.001\ 159\ 652\ 180\ 59\ (13)$

This is one of the most precisely known numbers in Physics and one that shows exceptional agreement with the experimentally measured value.

Toichiro Kinoshita spent his entire life on refining and improving the $a = \frac{g-2}{2}$ calculations. His career started at the University of Tokyo under S. Tomonaga. When Oppenheimer asked Tomonaga to recommend two promising graduate students to come to Princeton, Tomonaga chose Kinoshita and Y. Nambu. At Princeton, Kinoshita used to take a bus to take him to the University. A regular fellow passenger was Einstein. One day Einstein sat next to Kinoshita and asked him what he was working on. Quantum electrodynamics was the answer. Silence followed! On another trip, Einstein asked Nambu what he was working on. The theory of relativity said Nambu; Einstein sat with Nambu on many occasions in the future! After a brief stay at Columbia University in New York Kinoshita found himself on the faculty at Cornell University in Ithaca New York. He remained there for the rest of his career. Kinoshita's office was next to Feynman's. Someone had given both a difficult problem and Kinoshita and Feynman took it as an informal competition between the two of them. Kinoshita started working on the problem in his usual methodical way while Feynman next door was learning to play the bongos practicing by banging on the radiators irritating the living daylights of Kinoshita. (Both got the correct answer.)

Meticulous, persistent, careful, dedicated, Kinoshita became synonymous with impossibly difficult, high precision calculations.

Current work involves calculations on the anomalous magnetic moment of the muon. Unlike the case of the electron, where there is excellent agreement between theory and experiment, there appears to be a discrepancy in the case of the muon. This could be a sign that new unknown physics is in play, an exciting possibility!